XI
continued

Summary of Some of the Simple Harmonics

FIRST ORDER
(PI RATIOS)

Evaluation at π/2

Normalized Simple Series (First Piece)	Reciprocal of Normalized Series (First Piece)	Normalized Simple Series (Second Piece)	Reciprocal of Normalized Series (Second Piece)
1	1	1.4142	0.7071
-0.7071	-1.4142	-1	-1
0.3333	3	0.4714	2.1213
0	$7.6989\cdot10^{15}$	0	
-0.2	-5	-0.2828	-3.5355
0.2357	4.2426	0.3333	3
-0.1429	-7	-0.202	-4.9497
0	$-7.6989\cdot10^{15}$	0	

$$\frac{\pi}{4} = \sqrt{2}\left(1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\ldots\right)\cdot\left(1-\frac{1}{3}+\frac{1}{5}-\ldots\right)$$

$$\frac{\pi}{4} = \sum_{k=1}^{\infty}\left[\sqrt{2}\left[\frac{(-1)^{k-1}}{4k-3}+\frac{(-1)^{k-1}}{4k-1}\right]\cdot\frac{(-1)^{k-1}}{(2k-1)}\right] \qquad \frac{\pi}{4}=0.7854 \qquad \sum_{k=1}^{10000}\left[\sqrt{2}\left[\frac{(-1)^{k-1}}{4k-3}+\frac{(-1)^{k-1}}{4k-1}\right]\cdot\frac{(-1)^{k-1}}{(2k-1)}\right]=0.7854$$

SEMI-FIRST ORDER
(PI RATIO CONSTANT BASED)

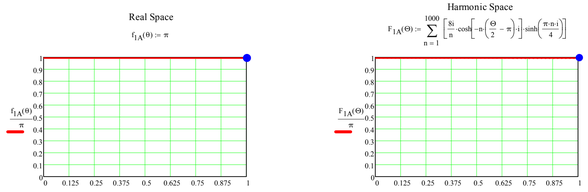
These maps are constant with both $\theta=\pi$ and $\Theta=\pi$ and are the derivatives of the second order maps to the right. If the Real and Harmonic Space maps are graphically equivalent then their derivatives must be too.

$$\frac{d}{d\theta}(\theta\cdot\pi)=\pi \text{ (Real)} \qquad \frac{d}{d\Theta}\left[\frac{16}{n+1}\cdot\sin\left[i\cdot n\left(\pi-\frac{\Theta}{2}\right)\right]\cdot\left[\frac{1}{n+1}\cdot\sin\left[i\cdot n\left(\frac{\pi}{4}\right)\right]\right]\right]=\frac{8i}{n}\cdot\cos\left[-n\left(\frac{\Theta}{2}-\pi\right)\cdot i\right]\cdot\sin\left(\frac{\pi\cdot n\cdot i}{4}\right) \text{ (Harmonic)}$$

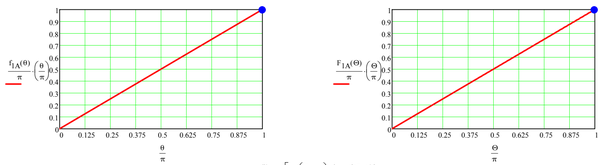
Evaluation at π

Real Space $f_{1A}(\theta):=\pi$

Harmonic Space $F_{1A}(\Theta):=\sum_{n=1}^{1000}\left[\frac{8i}{n}\cdot\cosh\left[-n\left(\frac{\Theta}{2}-\pi\right)\cdot i\right]\cdot\sinh\left(\frac{\pi\cdot n\cdot i}{4}\right)\right]$



Regraphed Linear for Comparison with Higher Orders



$$\theta = \frac{8i}{n}\cdot\cosh\left[-n\left(\frac{\pi}{2}-\pi\right)\cdot i\right]\cdot\sinh\left(\frac{\pi\cdot n\cdot i}{4}\right) \text{ (Transform)}$$

(Real) (Harmonic)

Normalized Simple Series	Reciprocal of Normalized Series
0	
1	1
0	
0	
0	
-0.3333	-3
0	
0	

$\pi = 4x$

$$\pi = 4\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\right) \qquad \frac{\pi}{4}=\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\right)=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)} \text{ (No new information)}$$

Since $\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)}=\frac{\pi}{4}$ then $\frac{\pi}{2}=\sum_{k=1}^{\infty}\left[\sqrt{2}\left[\frac{(-1)^{k-1}}{4k-3}+\frac{(-1)^{k-1}}{4k-1}\right]\right]$

So $\frac{\pi}{2\sqrt{2}}=\left(1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\ldots\right)=\sum_{k=1}^{\infty}\left(\frac{1}{8k-7}+\frac{1}{8k-5}-\frac{1}{8k-3}-\frac{1}{8k-1}\right)=\sum_{k=1}^{\infty}\left[\frac{(-1)^{k-1}}{4k-3}+\frac{(-1)^{k-1}}{4k-1}\right]$

$\frac{\pi}{2\sqrt{2}}=1.1107$ $\sum_{k=1}^{10000}\left(\frac{1}{8k-7}+\frac{1}{8k-5}-\frac{1}{8k-3}-\frac{1}{8k-1}\right)=1.1107$ $\left[\sum_{k=1}^{10000}\left[\frac{(-1)^{k-1}}{4k-3}+\frac{(-1)^{k-1}}{4k-1}\right]\right]=1.1107$

Now $\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\right)+\left(1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\ldots\right)=2\left(1+0-0-\frac{1}{7}+\ldots\right)$ and $\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\right)-\left(1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\ldots\right)=-2\left(0+\frac{1}{3}-\frac{1}{5}-0+\ldots\right)$

So $\frac{\pi}{4}+\frac{\pi}{2\sqrt{2}}=2\left[\sum_{k=1}^{\infty}\left(\frac{1}{8k-7}-\frac{1}{8k-1}\right)\right]$ and $\frac{\pi}{4}-\frac{\pi}{2\sqrt{2}}=-2\left[\sum_{k=1}^{\infty}\left(\frac{1}{8k-5}-\frac{1}{8k-3}\right)\right]$

Or $\frac{\pi}{8}\left(1+\sqrt{2}\right)=\sum_{k=1}^{\infty}\left(\frac{1}{8k-7}-\frac{1}{8k-1}\right)=\left(1-\frac{1}{7}+\frac{1}{9}-\frac{1}{15}+\ldots\right)$ and $\frac{\pi}{8}\left(\sqrt{2}-1\right)=\sum_{k=1}^{\infty}\left(\frac{1}{8k-5}-\frac{1}{8k-3}\right)=\left(\frac{1}{3}-\frac{1}{5}+\frac{1}{11}-\frac{1}{13}+\ldots\right)$

$\frac{\pi}{8}\left(1+\sqrt{2}\right)=0.9481$ $\sum_{k=1}^{10000}\left(\frac{1}{8k-7}-\frac{1}{8k-1}\right)=0.9481$ $\frac{\pi}{8}\left(\sqrt{2}-1\right)=0.1627$ $\sum_{k=1}^{10000}\left(\frac{1}{8k-5}-\frac{1}{8k-3}\right)=0.1627$

The odd terms $\sum_{k=1}^{\infty}\left[\frac{(-1)^{k-1}}{4k-3}+\frac{(-1)^{k-1}}{4k-1}\right]$, $\sum_{k=1}^{\infty}\left(\frac{1}{8k-7}-\frac{1}{8k-1}\right)$ and $\sum_{k=1}^{\infty}\left(\frac{1}{8k-5}-\frac{1}{8k-3}\right)$ cannot be split apart and still have a closed form.

The individual components do not converge as the limit of k goes to ∞, similar to $\sum_{k=1}^{\infty}\frac{1}{k}$, and if we could separate these it would be possible algebraicly rearrange them to create a closed form for $\sum_{k=1}^{\infty}\frac{1}{k}$ which clearly cannot be done since it does not converge.

SECOND ORDER
(PI SQUARED RATIOS)

Evaluation at π/2

Normalized Simple Series	Reciprocal of Normalized Series
1	1
-0.5	-2
0.1111	9
0	
0.04	25
-0.0556	-18
0.0204	49
0	

$$\left(\frac{\pi}{2}\right)^2 = 8x$$

$$\left(\frac{\pi}{2}\right)^2 = 8\cdot\frac{1}{2}\left[\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right)-\left(\frac{2}{2^2}+\frac{2}{6^2}+\ldots\right)\right] = \sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}-\sum_{k=1}^{\infty}\frac{2}{(4k-2)^2}$$

But $\sum_{k=1}^{\infty}\frac{2}{(4k-2)^2}=\frac{1}{2}\left[\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\right]$ and since $\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}=\frac{\pi^2}{8}$

$\frac{\pi^2}{16}=\frac{1}{2}\left[\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\right]$ and $\boxed{\frac{\pi^2}{32}=\sum_{k=1}^{\infty}\frac{1}{(4k-2)^2}}$ (No new information)

Since $\frac{\pi^2}{8}=\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}=\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right)$ and $\frac{\pi^2}{12}=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k^2}=\left(1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}\ldots\right)$

then $\frac{\pi^2}{8}-\frac{\pi^2}{12}=\left(\frac{1}{2^2}+\frac{1}{4^2}+\ldots\right)$ or $\boxed{\frac{\pi^2}{24}=\sum_{k=1}^{\infty}\frac{1}{(2k)^2}}=\frac{1}{4}\left(\sum_{k=1}^{\infty}\frac{1}{k^2}\right)$ or $\boxed{\frac{\pi^2}{6}=\sum_{k=1}^{\infty}\frac{1}{k^2}}$

Since $\frac{\pi^2}{24}=\sum_{k=1}^{\infty}\frac{1}{(2k)^2}=\left(\frac{1}{2^2}+\frac{1}{4^2}+\ldots\right)$ and $\frac{\pi^2}{32}=\sum_{k=1}^{\infty}\frac{1}{(4k-2)^2}=\left(\frac{1}{2^2}+\frac{1}{6^2}+\ldots\right)$ then $\frac{\pi^2}{24}-\frac{\pi^2}{32}=\left(\frac{1}{4^2}+\frac{1}{8^2}+\ldots\right)$ and $\boxed{\frac{\pi^2}{96}=\sum_{k=1}^{\infty}\frac{1}{(4k)^2}}$

$\frac{\pi^2}{32}=0.3084$ $\sum_{k=1}^{10000}\frac{1}{(4k-2)^2}=0.3084$ $\frac{\pi^2}{6}=1.6449$ $\sum_{k=1}^{10000}\frac{1}{k^2}=1.6448$ $\frac{\pi^2}{24}=0.4112$ $\sum_{k=1}^{10000}\frac{1}{(2k)^2}=0.4112$ $\frac{\pi^2}{96}=0.1028$ $\sum_{k=1}^{10000}\frac{1}{(4k)^2}=0.1028$

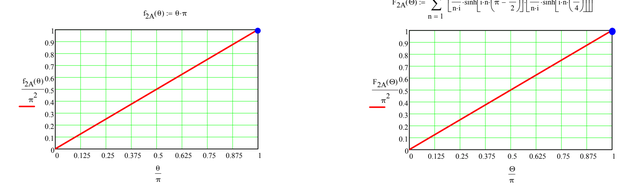
SEMI-SECOND ORDER
(PI SQUARED RATIO LINEAR BASED)

These maps are linear with θ and Θ.

Evaluation at π

Real Space $f_{2A}(\theta):=\theta\cdot\pi$

Harmonic Space $F_{2A}(\Theta):=\sum_{n=1}^{1000}\left[\frac{16}{n+1}\cdot\sin\left[i\cdot n\left(\pi-\frac{\Theta}{2}\right)\right]\cdot\left[\frac{1}{n+1}\cdot\sin\left[i\cdot n\left(\frac{\pi}{4}\right)\right]\right]\right]$



$$\theta\cdot\pi = \sum_{n=1}^{\infty}\left[\frac{16}{n+1}\cdot\sinh\left[i\cdot n\left(\pi-\frac{\Theta}{2}\right)\right]\cdot\left[\frac{1}{n+1}\cdot\sinh\left[i\cdot n\left(\frac{\pi}{4}\right)\right]\right]\right] \text{ (Transform)}$$

(Real) (Harmonic)

Normalized Simple Series	Reciprocal of Normalized Series
1	1
0	
-0.1111	-9
0	
-0.04	-25
0	
0.0204	49
0	

$$\pi^2 = \frac{16}{\sqrt{2}}\left(1-\frac{1}{3^2}-\frac{1}{5^2}+\frac{1}{7^2}+\ldots\right)$$

$$\frac{\pi^2}{8\sqrt{2}}=\left(1-\frac{1}{3^2}-\frac{1}{5^2}+\frac{1}{7^2}+\ldots\right)=\sum_{k=1}^{\infty}\left[\frac{1}{(8k-7)^2}-\frac{1}{(8k-5)^2}-\frac{1}{(8k-3)^2}+\frac{1}{(8k-1)^2}\right]=\sum_{k=1}^{\infty}\left[\frac{(-1)^{k-1}}{(4k-3)^2}-\frac{(-1)^{k-1}}{(4k-1)^2}\right]$$

$\frac{\pi^2}{8\sqrt{2}}=0.8724$ $\sum_{k=1}^{10000}\left[\frac{1}{(8k-7)^2}-\frac{1}{(8k-5)^2}-\frac{1}{(8k-3)^2}+\frac{1}{(8k-1)^2}\right]=0.8724$ $\sum_{k=1}^{10000}\left[\frac{(-1)^{k-1}}{(4k-3)^2}-\frac{(-1)^{k-1}}{(4k-1)^2}\right]=0.8724$

Since $\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}=\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\right)=\frac{\pi^2}{8}$ then

$\frac{\pi^2}{8}+\frac{\pi^2}{8\sqrt{2}}=\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\right)+\left(1-\frac{1}{3^2}-\frac{1}{5^2}+\frac{1}{7^2}+\ldots\right)=2\left(1-0-0+\frac{1}{7^2}+\ldots\right)$

Or $\boxed{\left(1+\frac{1}{\sqrt{2}}\right)\frac{\pi^2}{16}=\left(1+\frac{1}{7^2}+\ldots\right)=\sum_{k=1}^{\infty}\left[\frac{1}{(8k-7)^2}+\frac{1}{(8k-1)^2}\right]}$

Also $\frac{\pi^2}{8}-\frac{\pi^2}{8\sqrt{2}}=\left(1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\right)-\left(1-\frac{1}{3^2}-\frac{1}{5^2}+\frac{1}{7^2}+\ldots\right)=2\left(0+\frac{1}{3^2}+\frac{1}{5^2}+0+\ldots\right)$

So $\boxed{\left(1-\frac{1}{\sqrt{2}}\right)\frac{\pi^2}{16}=\left(\frac{1}{3^2}+\frac{1}{5^2}+\ldots\right)=\sum_{k=1}^{\infty}\left[\frac{1}{(8k-5)^2}+\frac{1}{(8k-3)^2}\right]}$

$\left(1+\frac{1}{\sqrt{2}}\right)\frac{\pi^2}{16}=1.053$ $\sum_{k=1}^{10000}\left[\frac{1}{(8k-7)^2}+\frac{1}{(8k-1)^2}\right]=1.053$ $\left(1-\frac{1}{\sqrt{2}}\right)\frac{\pi^2}{16}=0.1807$ $\sum_{k=1}^{10000}\left[\frac{1}{(8k-5)^2}+\frac{1}{(8k-3)^2}\right]=0.1807$

The odd terms $\sum_{k=1}^{\infty}\left[\frac{(-1)^{k-1}}{(4k-3)^2}-\frac{(-1)^{k-1}}{(4k-1)^2}\right]$, $\sum_{k=1}^{\infty}\left[\frac{1}{(8k-7)^2}+\frac{1}{(8k-1)^2}\right]$ and $\sum_{k=1}^{\infty}\left[\frac{1}{(8k-5)^2}+\frac{1}{(8k-3)^2}\right]$ do not appear to be able to be split apart and still have a closed form even though the individual components converge as the limit of k goes to ∞.

If true there is no closed form for $\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)^2}$ as it could be algebraically calculated from these components.

THIRD ORDER
(PI CUBED RATIOS)

Evaluation at π/2

Normalized Simple Series (First Piece)	Normalized Simple Series (Second Piece)	Normalized Simple Series (Third Piece)
1	1.65685	37.05974
-0.60355	-1	-22.36753
0.21587	0.35766	8
0	0	0
-0.04663	-0.07725	-1.728
0.02235	0.03704	0.82843
-0.00292	-0.00483	-0.10805
0	0	0

$$\left(\frac{\pi}{2}\right)^3 = 32\sqrt{2}\left(\sin\left(\frac{\pi}{8}\right)\right)^2 x$$

Reciprocal	Reciprocal	Reciprocal
1	0.60355	5.82843
-1.65685	-1	-9.65685
4.63247	2.79594	27
-21.44661	-12.94417	-125
44.73506	27	260.73506
-343	-207.01881	-1999.1505

$$\frac{\pi^3}{8} = 32\sqrt{2}\left(\sin\left(\frac{\pi}{8}\right)\right)^2\left(1-\frac{1}{7^3}+\frac{1}{9^3}-\frac{1}{15^3}+\ldots\right)-4\left(1-\frac{1}{3^3}+\frac{1}{5^3}-\ldots\right)+32\sqrt{2}\left(\sin\left(\frac{3\pi}{8}\right)\right)^2\left(\frac{1}{3^3}-\frac{1}{5^3}+\frac{1}{11^3}-\frac{1}{13^3}+\ldots\right)$$

so $\frac{\pi^3}{64}=\sum_{k=1}^{\infty}\left[4\sqrt{2}\left(\sin\left(\frac{\pi}{8}\right)\right)^2\left[\frac{1}{(8k-7)^3}-\frac{1}{(8k-1)^3}\right]-\frac{1}{2}\left[\frac{(-1)^{k-1}}{(2k-1)^3}\right]+4\sqrt{2}\left(\sin\left(\frac{3\pi}{8}\right)\right)^2\left[\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}\right]\right]$

But $\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)^3}=\frac{\pi^3}{32}$ so $\frac{\pi^3}{64}+\left(\frac{1}{2}\right)\frac{\pi^3}{32}=\frac{\pi^3}{32}=\sum_{k=1}^{\infty}\left[4\sqrt{2}\left(\sin\left(\frac{\pi}{8}\right)\right)^2\left[\frac{1}{(8k-7)^3}-\frac{1}{(8k-1)^3}\right]+4\sqrt{2}\left(\sin\left(\frac{3\pi}{8}\right)\right)^2\left[\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}\right]\right]$

or $\boxed{\frac{\pi^3}{128\sqrt{2}}=\sum_{k=1}^{\infty}\left[\sin\left(\frac{\pi}{8}\right)^2\left[\frac{1}{(8k-7)^3}-\frac{1}{(8k-1)^3}\right]+\left(\sin\left(\frac{3\pi}{8}\right)\right)^2\left[\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}\right]\right]}$ $\frac{\pi^3}{128\sqrt{2}}=0.1713$ $\sum_{k=1}^{10000}\left[\sin\left(\frac{\pi}{8}\right)^2\left[\frac{1}{(8k-7)^3}-\frac{1}{(8k-1)^3}\right]+\left(\sin\left(\frac{3\pi}{8}\right)\right)^2\left[\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}\right]\right]=0.1713$

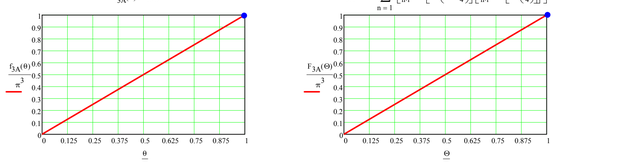
SEMI-THIRD ORDER
(PI CUBED RATIO LINEAR BASED)

These maps are linear with θ and Θ.

Evaluation at π

Real Space $f_{3A}(\theta):=\theta\cdot\pi^2$

Harmonic Space $F_{3A}(\Theta):=\sum_{n=1}^{1000}\left[\frac{64}{n+1}\cdot\sinh\left[i\cdot n\left(\pi-\frac{\Theta}{2}\right)\right]\cdot\left[\frac{\sqrt{2}}{n+1}\cdot\sinh\left[i\cdot n\left(\frac{\pi}{4}\right)\right]\right]^2\right]$



$$\theta\cdot\pi^2 = \sum_{n=1}^{\infty}\left[\frac{32}{n+1}\cdot\sinh\left[i\cdot n\left(\pi-\frac{\Theta}{2}\right)\right]\cdot\left[\frac{\sqrt{2}}{n+1}\cdot\sinh\left[i\cdot n\left(\frac{\pi}{4}\right)\right]\right]^2\right] \text{ (Transform)}$$

(Real) (Harmonic)

Normalized Simple Series (First Piece)	Reciprocal of Normalized Series (First Piece)	Normalized Simple Series (Second Piece)	Reciprocal of Normalized Series (Second Piece)
1	1	0.35355	2.82843
-0.35355	-2.82843	-0.125	-8
0.03704	27	0.01309	76.36753
0		0	
-0.008	-125	-0.00283	-353.55339
0.01309	76.36753	0.00463	216
-0.00292	-343	-0.00103	-970.1505
0		0	

$\pi^3 = \frac{64}{\sqrt{2}}x$ $\frac{\pi^3}{2} = 64x$

$$\pi^3 = \frac{64}{\sqrt{2}}\left(1+\frac{1}{3^3}-\frac{1}{5^3}-\frac{1}{7^3}+\ldots\right)-128\left(\frac{1}{3^3}-\frac{1}{6^3}+\ldots\right) \qquad \boxed{\frac{\sqrt{2}}{64}\pi^3=\sum_{k=1}^{\infty}\left[\frac{1}{(8k-7)^3}-\frac{2-\sqrt{2}}{(8k-6)^3}+\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}+\frac{2-\sqrt{2}}{(8k-2)^3}-\frac{1}{(8k-1)^3}\right]}$$

$\frac{\sqrt{2}}{64}\pi^3=0.6851$ $\sum_{k=1}^{10000}\left[\frac{1}{(8k-7)^3}-\frac{2-\sqrt{2}}{(8k-6)^3}+\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}+\frac{2-\sqrt{2}}{(8k-2)^3}-\frac{1}{(8k-1)^3}\right]=0.6851$

But $\sum_{k=1}^{\infty}\left[\frac{1}{(8k-6)^3}-\frac{1}{(8k-2)^3}\right]=\frac{1}{8}\left[\sum_{k=1}^{\infty}\left[\frac{1}{(4k-3)^3}-\frac{1}{(4k-1)^3}\right]\right]=\frac{1}{8}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)^3}=\frac{1}{8}\frac{\pi^3}{32}=\frac{\pi^3}{256}$

So $\frac{\sqrt{2}}{64}\pi^3+2\sqrt{2}\frac{\pi^3}{256}=\boxed{\frac{3\sqrt{2}}{128}\pi^3=\sum_{k=1}^{\infty}\left[\frac{1}{(8k-7)^3}+\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}-\frac{1}{(8k-1)^3}\right]=\sum_{k=1}^{\infty}\left[\frac{(-1)^{k-1}}{(4k-3)^3}+\frac{(-1)^{k-1}}{(4k-1)^3}\right]}$ (1)

$\frac{3\sqrt{2}}{128}\pi^3=1.0277$ $\sum_{k=1}^{10000}\left[\frac{1}{(8k-7)^3}+\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}-\frac{1}{(8k-1)^3}\right]=1.0277$ $\sum_{k=1}^{10000}\left[\frac{(-1)^{k-1}}{(4k-3)^3}+\frac{(-1)^{k-1}}{(4k-1)^3}\right]=1.0277$

Also $\frac{\pi^3}{32}=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)^3}=\sum_{k=1}^{\infty}\left[\frac{1}{(8k-7)^3}-\frac{1}{(8k-5)^3}+\frac{1}{(8k-3)^3}-\frac{1}{(8k-1)^3}\right]$ (2)

By adding (1) + (2) and dividing by 2 $\frac{\frac{\pi^3}{32}+\frac{3\sqrt{2}}{128}\pi^3}{2}=\boxed{\left(1+\frac{3\sqrt{2}}{4}\right)\frac{\pi^3}{64}=\sum_{k=1}^{\infty}\left[\frac{1}{(8k-7)^3}-\frac{1}{(8k-1)^3}\right]}$

By subtracting (1) - (2) and dividing by 2 $\frac{\frac{\pi^3}{32}-\frac{3\sqrt{2}}{128}\pi^3}{2}=\boxed{\left(\frac{3\sqrt{2}}{4}-1\right)\frac{\pi^3}{64}=\sum_{k=1}^{\infty}\left[\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}\right]}$

$\left(1+\frac{3\sqrt{2}}{4}\right)\frac{\pi^3}{64}=0.9983$ $\sum_{k=1}^{10000}\left[\frac{1}{(8k-7)^3}-\frac{1}{(8k-1)^3}\right]=0.9983$ $\left(\frac{3\sqrt{2}}{4}-1\right)\frac{\pi^3}{64}=0.0294$ $\sum_{k=1}^{10000}\left[\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}\right]=0.0294$

The odd terms $\sum_{k=1}^{\infty}\left[\frac{(-1)^{k-1}}{(4k-3)^3}-\frac{(-1)^{k-1}}{(4k-1)^3}\right]$, $\sum_{k=1}^{\infty}\left[\frac{1}{(8k-7)^3}-\frac{1}{(8k-1)^3}\right]$ and $\sum_{k=1}^{\infty}\left[\frac{1}{(8k-5)^3}-\frac{1}{(8k-3)^3}\right]$ do not appear to be able to be split apart and still have a closed form even though the individual components converge as the limit of k goes to ∞.

If true there is no closed form for $\sum_{k=1}^{\infty}\frac{1}{(2k-1)^3}$ as it could be algebraically calculated from these components.