A. Introduction

The fundamental constants e and π are considered basic and cannot be derived. Nature, however, is tasked with providing these values and a means of implementation. This mechanism has proven elusive because insufficient variables exist in real space and these fundamental constants are unit-less and transparent.

We describe (a mapping technique involving) an underlying virtual space which is rich in structure and provides the necessary variables to maintain natural order. It is composed of waves harmonically ordered similar to the Fourier Transform.

Harmonic space is married to real space through an equivalence relation which requires that the two observation methods agree on the result of a common event. For each fundamental constant the equivalence relation has a specific solution set. It contains the coding pattern for combining the individual harmonics for equivalence. Each combination establishes a specific harmonic mode. Specific modes correspond to the fundamental constants and determine their value.

This paper derives the fundamental constants e and π from first principles using this equivalence relation and describes the method of implementation. The procedure is expanded to include the Fibonacci Series.

B. Procedure

This is the general procedure. First we set up an equivalence relation called the General Transformation Equation or GTE. In its most general form:

GTE $$\int_A^B f(x)dx = \int_A^B F(X)dX$$

The left hand side (LHS) of the equation represents an action $A \rightarrow B$ in real space x. The right hand side (RHS) represents the mapping of this same event in harmonic space X. Since they both describe the same event they must agree on the result. Hence the integral of the mapping function in one space must equal the integral of the mapping function in the other space.

The harmonic mapping function $F(X)$ and the boundary conditions A and B must be properly chosen for equivalence. Typically we graph the left hand side in x space and the right hand side in X space to demonstrate they are graphically equivalent over the entire range.

Harmonic space is made up of an infinite series of wave functions. When the right hand side is evaluated at the boundary conditions it will define a specific solution set. Since the two sides are the results of alternative mappings of the same even they must agree on the result. This establishes a relationship between the fundamental constant in real space and the solution set in harmonic space. Not only does it define the constant's value, it also provides the underlying mechanics that require the constant to have this value.

C. Similarity to the Fourier Transform

Our approach is similar to the Fourier transform. In Fourier analysis a periodic signal, which by nature varies in time, is expressed as the sum of component waves. The component waves are numbered and vary in frequency and amplitude. When these individual waves are added together they result in the original signal. We view this mathematical transform as a remapping of the signal from one space to another (time to frequency). We use the term "space" liberally here to mean any variable we can map our data onto including time and frequency. The importance of this is that a signal which appears highly irregular and mathematically unmanageable in the frame we are accustomed to viewing it, in this case it is time, is more naturally expressed in terms of frequency.

Fourier showed that a periodic function can be expressed exactly, or "transformed", into the sum of a series of sine or cosine wave functions. See Figures 1A & 1

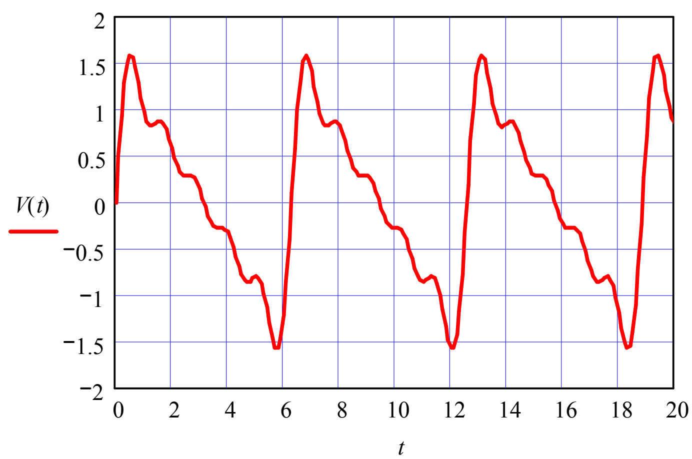

Figure 1A

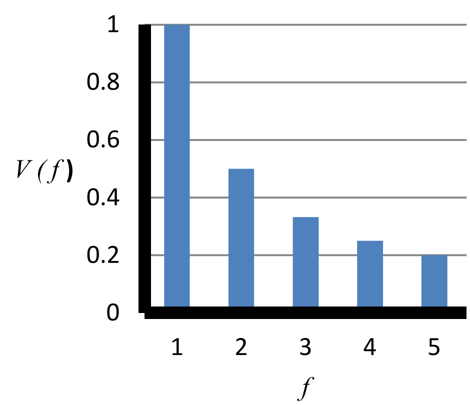

Figure1B

In Figure 1A $V(t) = \sin(t) + \frac{1}{2}\sin(2t) + \frac{1}{3}\sin(3t) + \frac{1}{4}\sin(4t) + \frac{1}{5}\sin(5t)$ is displayed in time space, and in Figure 1B the same function is mapped in frequency space as $V(f)$. We see that $V(t)$ appears to be very erratic when graphed in time space. Yet it is just the sum of five simple waves of progressively finer frequency and smaller amplitude. $V(f)$ indicates the amplitude of each of the five components $V_1(t)$ through $V_5(t)$ that add to equal $V(t)$.

Figure 1B is somewhat analogous to a key. A real key is cut in a specific pattern so that each pin in the lock is raised to a specific elevation to open the lock. In the same way $V(f)$ indicates the amplitude of each component wave that is needed. It is a key that contains the coding pattern needed for equivalence between the mapping spaces.

Figure 1C shows the five component waves and their sum, *V(t)*. Note that the components have progressively smaller amplitude and higher frequency and that adding the component waves produces a function equivalent to *V(t)*. The steep initial rise and spike is due to all the components increasing in this interval. The smaller humps are due to the higher frequency and smaller amplitude components. Lastly, all of the components have a value of zero at multiples of π and therefore *V(t)* must go to zero here as well.

Our approach is similar to this. An unexplainable natural constant like e or π is found to model well as the sum of a series of ordered wave forms under certain boundary conditions. A specific amount of each individual component wave is needed for this to occur, analogous to a key. There is a coding pattern that determines all of these amounts. Evaluating the right hand side of the GTE at the boundary conditions determines this pattern. The GTE also demands that the results from the two mapping systems agree. In this way the fundamental constant in real space relates directly to the coding pattern in harmonic space.

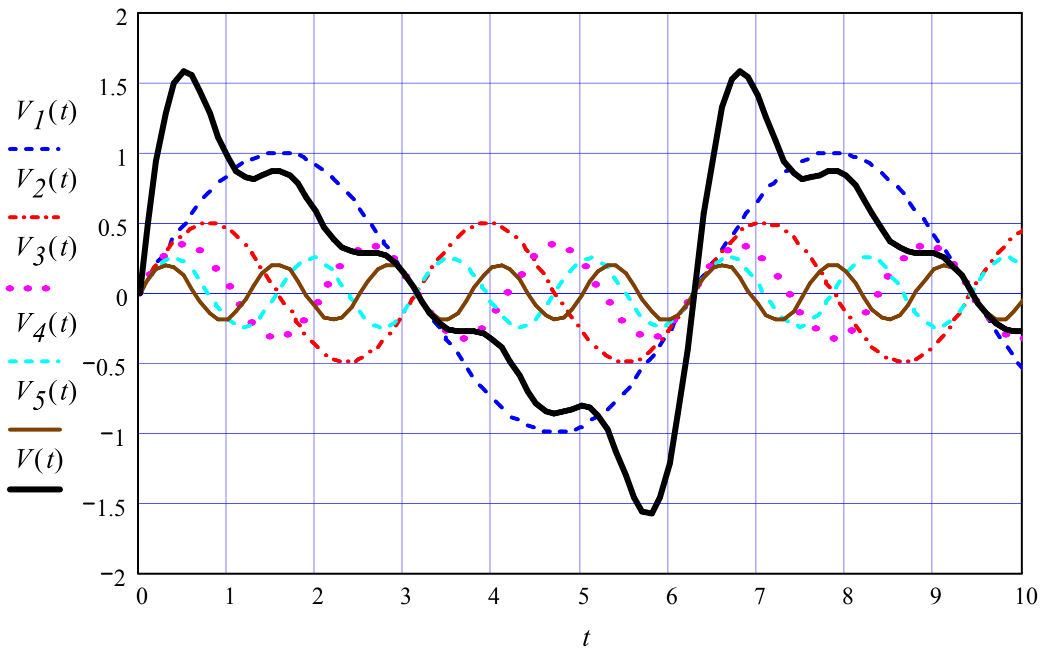

Figure 1C – The five component waves $V_1(t)$ thru $V_5(t)$ add to form $V(t)$

D. <u>The Five Spring Theatre</u>

This analogy provides a physical example of observing the same phenomena from different spaces. We have an audience in a theatre viewing a screen which is projected at from behind with a bouncing light. See Figure 2. The audience sees a light which bounces up and down in an intriguing but erratic way but they cannot see the device which moves it. It is a complex motion and the viewers cannot picture any simple device that would do this. If we were to put this light on cart and roll it across the stage it might make the pattern of a local mountain or building or some other complex design. We would need some sort of translucent and phosphorescent screen so that the part which has already been “drawn” does not disappear. The audience’s perspective is that of time space and they cannot view frequency space.

Behind the curtain is a stagehand. He sees the light as a small rod suspended from five springs of different size and frequency that are connected to each other, the largest at the top and the smallest at the bottom. From here it does not look mysterious. Each spring simply bounces at its own natural rhythm. To the stagehand the system moves in a natural way. He doesn’t see the complex light movements and cannot appreciate the pattern of a nearby mountain created as the cart and bouncing light roll across the stage. To him there are five connected springs vibrating in a normal way on a rolling cart and the magic and mystery are gone. The stagehand's perspective is that of frequency space.

This demonstrates how viewers in a different space might view the same phenomena. The audience in the theatre sees a complex motion in time which they cannot explain. The stagehand behind the screen sees the motion as logically based on mechanics and frequencies, but can't see the pattern.

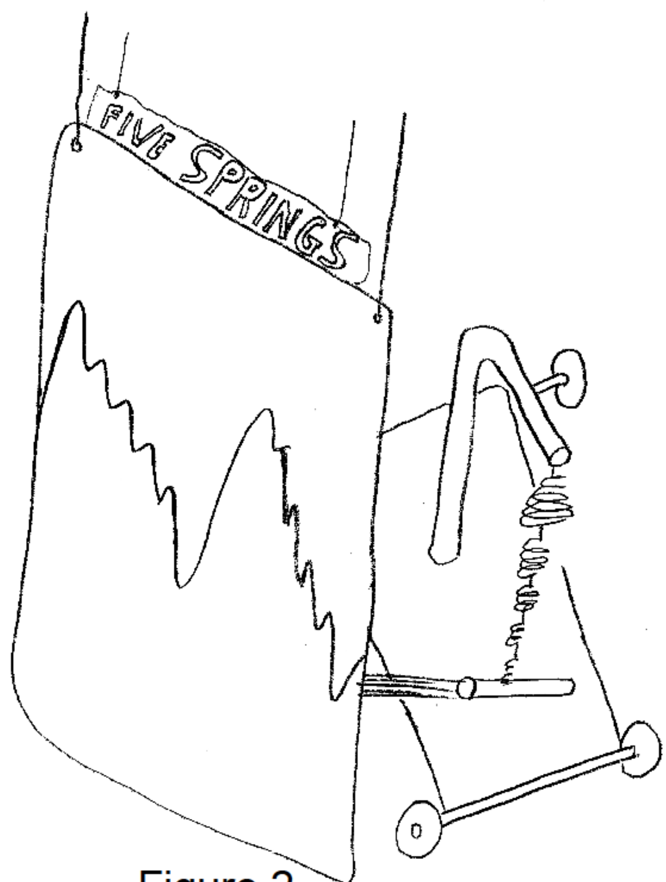

Figure 2

This brings us to Modal Harmonic Analysis or MHA. ***<u>Since both viewers are observing the same physical event, the result that each arrives at from their different mappings must be in agreement for that common physical event.</u>*** In our example the movements of the individual springs on a rolling cart must add up to produce the intriguing pattern the audience sees.

This is not necessarily intuitive despite the simplicity of the words. Equating different spaces is like equating apples and oranges. Does it require us to believe that apples might equal oranges? No. People in apple space must count with apples and those in orange space must use oranges. They just have to agree on what actually happened. Mathematically we would say that the integral within apple space equal must equal the integral within orange space. That is the basis of this analysis.