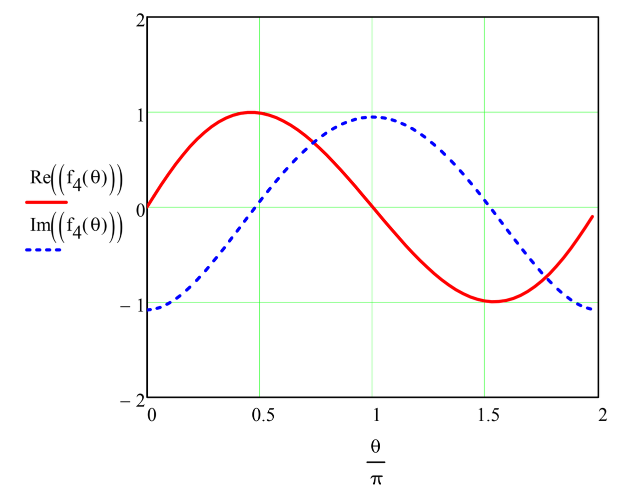
Casey's Curves

by Michael Ehrlich www.gte-harmonics.com fractal@cmn.net

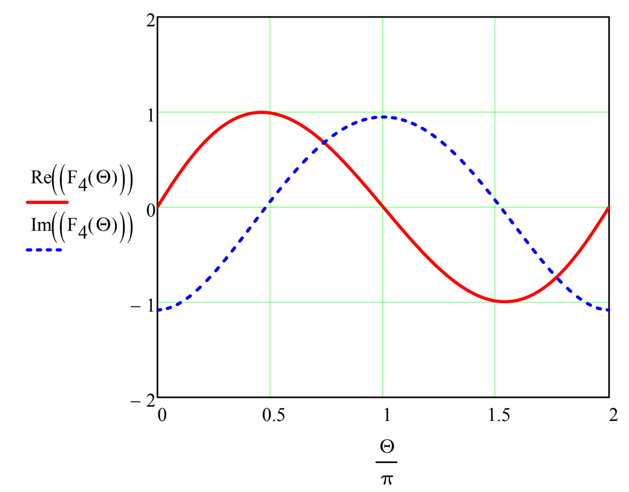
FOURTH ORDER

$$f_4(\theta) = \int_0^{\theta}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\cdot C_4 \qquad F_4(\Theta) = -i\sum_{n=1}^{\infty}\frac{e^{i\cdot n\cdot\Theta}}{n^4}$$

REAL SPACE



HARMONIC SPACE



$f_4(0) = -1.082i \quad f_4\left(\frac{\pi}{2}\right) = 0.989 + 0.059i \quad f_4(\pi) = 0.947i$

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FOURTH ORDER

Integrate the third order functions to obtain the fourth order:

Real Space

$$\int f_3(\theta)\,d\theta = \int\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta = \int_0^{\theta}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\cdot C_4$$

with $C_4 = \frac{8}{15}\int_0^{\pi}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta = \frac{\pi^4}{90}$

Harmonic Space

$$\int F_3(\Theta)\,d\Theta = \int\sum_{n=1}^{\infty}\frac{e^{i\cdot n\cdot\Theta}}{n^3}\,d\Theta = -i\sum_{n=1}^{\infty}\frac{e^{i\cdot n\cdot\Theta}}{n^4}$$

Then the fourth order mapping relationship is

$$\underbrace{\int_0^{\theta}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\frac{\pi^4}{90}}_{\text{Real Space}} = \underbrace{-i\sum_{n=1}^{\infty}\frac{e^{i\cdot n\cdot\Theta}}{n^4}}_{\text{Harmonic Space}}$$

Evaluate at 0

$$\int_0^{0}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\frac{\pi^4}{90} =$$

$$\frac{-i\cdot e^{i\cdot n\cdot 0}}{n^4} = \begin{pmatrix}-1.000i\\-0.063i\\-0.012i\\-0.004i\\-0.002i\\-0.001i\\-0.000i\\-0.000i\end{pmatrix} = -i\sum_{n=1}^{\infty}\frac{1}{n^4} = -i\,\mathrm{Li}_4(1)$$

$$\int_0^{0}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\frac{\pi^4}{90} = -1.082i$$

Evaluate at $\frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\frac{\pi^4}{90} =$$

$$\frac{-i\cdot e^{i\cdot n\cdot\frac{\pi}{2}}}{n^4} = \begin{pmatrix}1.000\\0.063i\\-0.012\\-0.004i\\0.002\\0.001i\\-0.000\\-0.000i\end{pmatrix} = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^4} + i\cdot\frac{1}{16}\cdot\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^4} = -i\,\mathrm{Li}_4(i)$$

$$\int_0^{\frac{\pi}{2}}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\frac{\pi^4}{90} = 0.989 + 0.059i$$

Evaluate at π

$$\int_0^{\pi}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\frac{\pi^4}{90} =$$

$$\frac{-i\cdot e^{i\cdot n\cdot\pi}}{n^4} = \begin{pmatrix}1.000i\\-0.063i\\0.012i\\-0.004i\\0.002i\\-0.001i\\0.000i\\-0.000i\end{pmatrix} = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^4} = -i\,\mathrm{Li}_4(-1)$$

$$\int_0^{\pi}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\frac{\pi^4}{90} = 0.947i$$

Only the Real Component is needed, the Imaginary is redundant:

But $\int_0^{0}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta = 0$

So $\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90} = \mathrm{Li}_4(1) = 1.082..$

$$\mathrm{Re}\int_0^{\frac{\pi}{2}}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\frac{\pi^4}{90} = \frac{1}{2}\cdot\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta\,d\theta\,d\theta + \frac{\pi}{2}\cdot C_3$$

$$= \mathrm{Re}\;-i\sum_{n=1}^{\infty}\frac{e^{i\cdot n\cdot\Theta}}{n^4} = \frac{\mathrm{Li}_4(i)-\mathrm{Li}_4(-i)}{2\cdot i}$$

So $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^4} = \frac{1}{2}\cdot\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta\,d\theta\,d\theta + \frac{\pi}{2}\cdot C_3 = \frac{\mathrm{Li}_4(i)-\mathrm{Li}_4(-i)}{2\cdot i} = 0.989...$

But $\int_0^{\pi}\int_0^{\theta}\int_0^{\theta}\ln\left(1-e^{i\cdot\theta}\right)d\theta + i\cdot\frac{\pi^2}{6}\,d\theta + C_3\,d\theta - i\frac{\pi^4}{90} = i\frac{\pi^4}{48} - i\frac{\pi^4}{90} = i\frac{7}{720}\pi^4$

So $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^4} = \frac{7}{720}\pi^4 = -\mathrm{Li}_4(-1) = 0.947..$

And $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} = \frac{15}{16}\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{i}{24}\cdot\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\pi}\ln\left(1-e^{i\cdot\theta}\right)d\theta\,d\theta\,d\theta$

So $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} = \frac{\pi^4}{96} = \frac{15}{16}\cdot C_4 = \frac{15}{16}\cdot\mathrm{Li}_4(1) = 1.015..$