Casey's Curves

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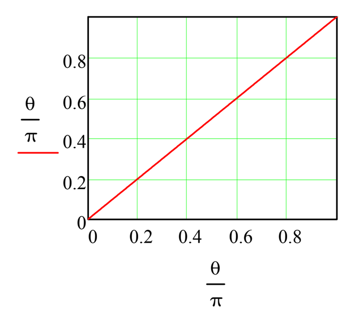
Note how with increasing order the pi power increases and alternates between the *alternating odd series* and the *alternating all series*, and the harmonic space mapping function alternates between sine & cosine functions. This is because when evaluated at Θ = π, the sine at this value generates the *alternating odd series* and the cosine generates the *alternating all series*, and successive iteration of integrals iteration result in alternation between sine and cosine (and periodic sign changes).

USING THE METHOD OF INTEGRATION TO DETERMINE HIGHER ORDER MAPPING FUNCTIONS

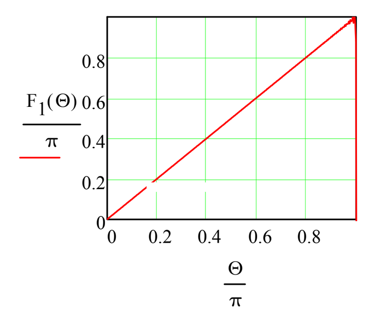
FIRST ORDER

$$f_1(\theta) = \theta \qquad F_1(\Theta) = \sum_{n=1}^{\infty} \frac{2\cdot\sin[n\cdot(\pi-\Theta)]}{n}$$

REAL SPACE



HARMONIC SPACE



Real Space Harmonic Space

Use first order mapping functions: $f_1(\theta) = \theta \qquad F_1(\Theta) = \sum_{n=1}^{\infty} \frac{2\cdot\sin[n\cdot(\pi-\Theta)]}{n}$

Then the first order mapping relationship is $\theta = \sum_{n=1}^{\infty} \frac{2\cdot\sin[n\cdot(\pi-\Theta)]}{n}$

Real Space Harmonic Space

Or: $\frac{1}{2}\cdot\theta = \sum_{n=1}^{\infty} \frac{\sin[n\cdot(\pi-\Theta)]}{n}$

Evaluate Harmonic Space function at π/2 to obtain series

$$\frac{1}{2}\cdot\frac{\pi}{2} = \frac{\sin\left[n\cdot\left(\pi-\frac{\pi}{2}\right)\right]}{n} =$$

1
0
-0.333
0
0.2
0
-0.143
0

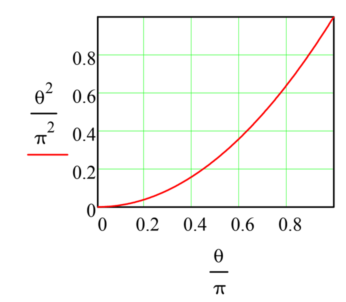
$$= 1 - \frac{1}{3} + \frac{1}{4}\ldots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$$

Therefore $\boxed{\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}}$ *Alternating Odd*

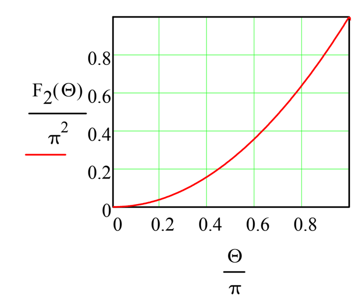
SECOND ORDER

$$f_2(\theta) = \theta^2 \qquad F_2(\Theta) = \sum_{n=1}^{\infty} \frac{4\cdot\cos[n\cdot(\pi-\Theta)]}{n^2} + \frac{\pi^2}{3}$$

REAL SPACE



HARMONIC SPACE



Integrate both the first order real Space and Harmonic Space mapping functions to obtain the second order functions. Because the first order functions are graphically equivalent the integrals of these must be as well.

Real Space Harmonic Space

$$\int \theta\, d\theta = \frac{\theta^2}{2} \qquad \int \sum_{n=1}^{\infty} \frac{2\sin[n\cdot(\pi-\Theta)]}{n}\, d\Theta = \sum_{n=1}^{\infty} \frac{2\cdot\cos[n\cdot(\pi-\Theta)]}{n^2} + C \qquad C = \frac{\pi^2}{6}$$

(C is determined by evaluation at Θ = 0)

The second order mapping relationship is: $\frac{\theta^2}{2} = \sum_{n=1}^{\infty} \frac{2\cdot\cos[n\cdot(\pi-\Theta)]}{n^2} + \frac{\pi^2}{6}$

Real Space Harmonic Space

Or: $\theta^2 = \sum_{n=1}^{\infty} \frac{4\cdot\cos[n\cdot(\pi-\Theta)]}{n^2} + \frac{\pi^2}{3}$

Evaluate HS Term at π/2

$$\frac{4\cdot\cos\left[n\cdot\left(\pi-\frac{\pi}{2}\right)\right]}{n^2} =$$

0
-1
0
0.25
0
-0.111
0
0.063

$$= -1 + \frac{1}{4} - \frac{1}{9}\ldots = -\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$$

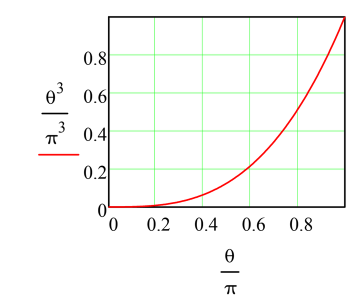
So $\left(\frac{\pi}{2}\right)^2 = -\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} + \frac{\pi^2}{3}$

Therefore $\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}}$ *Alternating All*

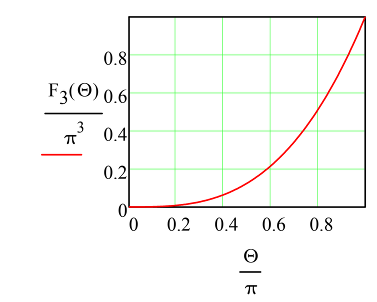
THIRD ORDER

$$f_3(\theta) = \theta^3 \qquad F_3(\Theta) = \sum_{n=1}^{\infty} \frac{12\cdot\sin[n\cdot(\Theta-\pi)]}{n^3} + \pi^2\cdot\Theta$$

REAL SPACE



HARMONIC SPACE



Integrate both the second order mapping functions to obtain the third order, and so on for higher orders..

Real Space Harmonic Space

$$\int \theta^2\, d\theta = \frac{\theta^3}{3} \qquad \int \left(\sum_{n=1}^{\infty} \frac{4\cdot\cos[n\cdot(\pi-\Theta)]}{n^2} + \frac{\pi^2}{3}\right) d\Theta = \sum_{n=1}^{\infty} \frac{-4\cdot\sin[n\cdot(\pi-\Theta)]}{n^3} + \frac{\pi^2}{3}\cdot\Theta + C \qquad C = 0$$

The third order mapping relationship is: $\frac{\theta^3}{3} = \sum_{n=1}^{\infty} \frac{-4\cdot\sin[n\cdot(\pi-\Theta)]}{n^3} + \frac{\pi^2}{3}\cdot\Theta$

Real Space Harmonic Space

Or: $\theta^3 = \sum_{n=1}^{\infty} \frac{-12\cdot\sin[n\cdot(\pi-\Theta)]}{n^3} + \pi^2\cdot\Theta$

Evaluate HS Term at π/2

$$\frac{\sin\left[n\cdot\left(\pi-\frac{\pi}{2}\right)\right]}{n^3} =$$

1
0
-0.037
0
0.008
0
-0.0029
0

$$= 1 - \frac{1}{27} + \frac{1}{125}\ldots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^3}$$

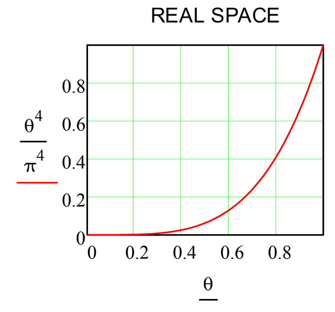
So $\left(\frac{\pi}{2}\right)^3 = -12\cdot\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^3} + \frac{\pi^3}{2}$

Therefore $\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^3} = \frac{\pi^3}{32}}$ *Alternating Odd*

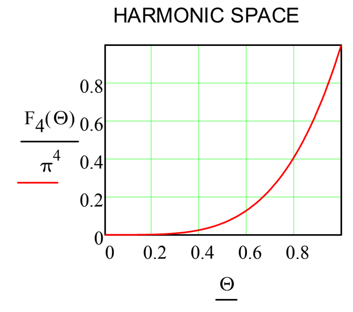
FOURTH ORDER

$$f_4(\theta) = \theta^4 \qquad F_4(\Theta) = 2\cdot\pi^2\cdot\Theta^2 - \sum_{n=1}^{\infty} \frac{48\cdot\cos[n\cdot(\pi-\Theta)]}{n^4} - \frac{7}{15}\cdot\pi^4$$

REAL SPACE



HARMONIC SPACE



Real Space Harmonic Space

$$\int \theta^3\, d\theta = \frac{\theta^4}{4} \qquad \int \left(\sum_{n=1}^{\infty} \frac{12\cdot\sin[n\cdot(\Theta-\pi)]}{n^3} + \pi^2\cdot\Theta\right) d\Theta = \sum_{n=1}^{\infty} \frac{-12\cdot\cos[n\cdot(\pi-\Theta)]}{n^4} + \frac{1}{2}\pi^2\cdot\Theta^2 + C \qquad C = -\frac{7}{60}\cdot\pi^4$$

The fourth order relationship is: $\frac{\theta^4}{4} = \frac{1}{2}\cdot\pi^2\cdot\Theta^2 - \sum_{n=1}^{\infty} \frac{12\cdot\cos[n\cdot(\pi-\Theta)]}{n^4} - \frac{7}{60}\cdot\pi^4$

Or: $\theta^4 = 2\cdot\pi^2\cdot\Theta^2 - \sum_{n=1}^{\infty} \frac{48\cdot\cos[n\cdot(\pi-\Theta)]}{n^4} - \frac{7}{15}\cdot\pi^4$

Evaluate HS Term at π/2

$$\frac{-16\cos\left[n\cdot\left(\pi-\frac{\pi}{2}\right)\right]}{n^4} =$$

0
1
0
-0.0625
0
0.0123
0
-0.0039

$$= 1 - \frac{1}{16} + \frac{1}{81}\ldots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4}$$

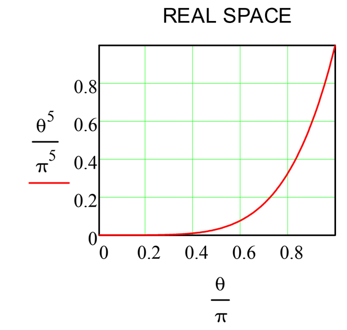
So $\left(\frac{\pi}{2}\right)^4 = 2\cdot\pi^4\cdot\left(\frac{\pi}{2}\right)^2 + 3\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4} - \frac{7}{15}\cdot\pi^4$

Therefore $\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4} = \frac{7}{720}\cdot\pi^4}$ *Alternating All*

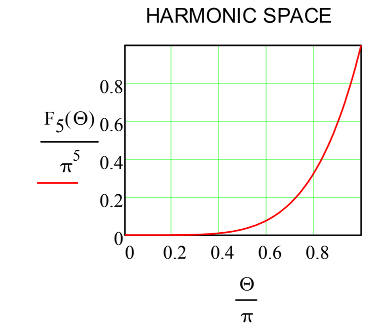
FIFTH ORDER

$$f_5(\theta) = \theta^5 \qquad F_5(\Theta) = \frac{10}{3}\cdot\pi^2\cdot\Theta^3 + \sum_{n=1}^{\infty} \frac{240\cdot\sin[n\cdot(\pi-\Theta)]}{n^5} - \frac{7}{3}\cdot\pi^4\cdot\Theta$$

REAL SPACE



HARMONIC SPACE



Real Space Harmonic Space

$$\int \theta^4\, d\theta = \frac{\theta^5}{5} \qquad \int \left(2\cdot\pi^2\cdot\Theta^2 - \sum_{n=1}^{\infty} \frac{48\cdot\cos[n\cdot(\pi-\Theta)]}{n^4} - \frac{7}{15}\cdot\pi^4\right) d\Theta = \frac{2}{3}\cdot\pi^2\cdot\Theta^3 + \sum_{n=1}^{\infty} \frac{48\cdot\sin[n\cdot(\pi-\Theta)]}{n^5} - \frac{7}{15}\cdot\pi^4\cdot\Theta + C \qquad C = 0$$

The fifth order relationship is: $\frac{\theta^5}{5} = \frac{2}{3}\cdot\pi^2\cdot\Theta^3 + \sum_{n=1}^{\infty} \frac{48\cdot\sin[n\cdot(\pi-\Theta)]}{n^5} - \frac{7}{15}\cdot\pi^4\cdot\Theta$

Or: $\theta^5 = \frac{10}{3}\cdot\pi^2\cdot\Theta^3 + \sum_{n=1}^{\infty} \frac{240\cdot\sin[n\cdot(\pi-\Theta)]}{n^5} - \frac{7}{3}\cdot\pi^4\cdot\Theta$

Evaluate HS Term at π/2

$$\frac{\sin\left[n\cdot\left(\pi-\frac{\pi}{2}\right)\right]}{n^5} =$$

1
0
-0.00412
0
0.00032
0
-0.00006
0

$$= 1 - \frac{1}{243} + \frac{1}{3125}\ldots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^5}$$

So $\left(\frac{\pi}{5}\right)^5 = \frac{10}{3}\cdot\pi^2\cdot\left(\frac{\pi}{2}\right)^3 + 240\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^5} - \frac{7}{3}\cdot\pi^4\cdot\frac{\pi}{2}$

Therefore $\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^5} = \frac{5}{1536}\cdot\pi^5}$ *Alternating Odd*